19. Prove that u and v be two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at a interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c. Then the function $f = u + iv$ has a derivative at c. Moreover $f'(c) = D_1 u(c) + iD\, v(c)$.

20. A quadric surface with center at the origin has the equation $Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$, Find the lengths of its semi-axes.

NOVEMBER/DECEMBER 2023

GMA22 — REAL ANALYSIS-II

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Lebesgue Integral.

2. Define Monotonic Sequences.

3. Define Riemann-integrable.

4. Define Measurable Functions.

5. Define Orthogonal Systems.

6. Define Dirichlet Integrals.

7. Directional Derivative.

8. Write down Taylor's formula.

9. Prove that $u + iv$ is a complex-valued function with a derivative at a point z in $C in\ f(z) = |f'(z)|^2$.

10. State Inverse Function theorem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11\. (a) Prove that $f \in U(I)$ and $g \in U(I)$, and if $f(x) = g(x)$ almost everywhere on I, Then $\int f = \int g$.

Or

(b) Prove that $f \in U(I)$ and $\{s_n\}$ and $\{t_m\}$ be two sequences generating f. Then $\lim_{n\to\infty} \int s_n = \lim_{n\to\infty} \int t_m$.

12\. (a) Prove that $f \in M(I)$ and if $|f(x)| \le g(x)$ almost everywhere on I for some nonnegative g in L(I), then $f \in L(I)$.

Or

(b) Prove that $f \in L(I)$ and if f is bounded almost everywhere on I, then $f^2 \in L(I)$.

13\. (a) Prove that g is of bounded variation on $[0, \delta]$, Then $\lim_{\alpha\to+\infty} \frac{2}{\pi}\int_0^\delta g(t)\frac{\sin \alpha t}{t} dt$ is equal to $g(0+)$.

Or

(b) Show that $x = \pi - 2\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ if $0 < x < 2\pi$.

14\. (a) Prove that S be an open connected subset of R^n, and let $f : S \to R^m$ be differentiable at each point at each point of S. If $f'(c) = 0$ for each c in S, Then f is constant on S.

Or

(b) State and Prove that Taylor's theorem.

15\. (a) Prove that A be an open subset of R^n and $f : A \to R^n$ is continuous and has finite partial derivative $D_i f_i$ on A. if f is one-to-one on A and if $J_f(X) \ne 0$ for each x in A, then $f(A)$ is open.

Or

(b) State and prove Implicit function theorem.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16\. State and Prove Lebesgue dominated convergence theorem.

17\. State and Prove Riesz- Fischer theorem.

18\. State and Prove Weierstrass approximation theorem.